Student Name : ______________________

Class Teacher :______________

HURLSTONE AGRICULTURAL HIGH SCHOOL

YEAR 12 2010

MATHEMATICS EXTENSION 1

TRIAL HIGHER SCHOOL CERTIFICATE

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General Instructions

- Reading time : 5 minutes
- **Working time : 2 hours**
- Attempt **all** questions
- **Start a new sheet of paper for each question**
- All necessary working should be shown
- This paper contains 8 questions worth 10 marks each. Total Marks: **80 marks**
- Marks may not be awarded for careless or badly arranged work
- Board approved calculators and mathematical templates may be used
- This examination paper must **not** be removed from the examination room

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2010 HSC Mathematics Extension 1 Examination.

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C, \quad n \neq -1; \; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x + C, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax + C, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax + C, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax + C, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax + C, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a} + C, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right) + C, \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right) + C$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Marks

QUESTION 1. *Start a new answer booklet.*

(a) Let $A(4,-1)$ and $B(-3,2)$ be points on the number plane. Find the coordinates of the point P which divides the interval AB internally in the ratio 3:2. **2**

(b) (i) Prove that $\cos 2x = 1 - 2\sin^2 x$. **1**

(ii) On the same diagram, sketch the curves $y = \cos 2x$ and $y = 2\sin^2 x$, for $0 \le x \le \pi$. **2**

(iii) Find the points of intersection of the two curves, in the domain $0 \le x \le \pi$. **2**

(iv) Determine the acute angle between the two curves at the point where $x = \frac{\pi}{6}$. **3**

QUESTION 2. *Start a new answer booklet.*

(a) Newton's Law of Cooling states that the rate of change in the temperature, T°, of a body is proportional to the difference between the temperature of the body and the surrounding temperature, P°.

(i) If A and k are constants, show that $T = P + Ae^{kt}$ satisfies the equation $\frac{dT}{dt} = k(T - P)$. **1**

(ii) A cup of tea with temperature 100°C is too hot to drink. If two minutes later, the temperature has dropped to 93°C and the surrounding temperature is 23°C, calculate A and k. **2**

(iii) How long, to the nearest minute, will it take for the tea to reach the drinkable temperature of 80°C? **2**

(b) A particles displacement x centimetres from O at time t seconds, is given by $x = 3\cos\left(2t + \frac{\pi}{3}\right)$.

(i) Express the acceleration as a function of displacement and hence show the the particle undergoes simple harmonic motion about the origin O. **3**

(ii) Find the value of x for which the speed is a maximum and determine this speed. **2**

Marks

QUESTION 3. *Start a new answer booklet.*

(a) Given $6^k - 1$ is divisible by 5 for all positive integral values of k, prove that $6^{k+1} - 1$ is also divisible by 5. **2**

(b) By the process of mathematical induction, prove the following true for all positive integers n: **3**

$$\sum_{r=1}^{n} r.2^r = (n-1).2^{n+1} + 2$$

(c) At a school prefect induction ceremony, 16 prefects (8 girls and 8 boys) were to be seated at the front of hall in two rows.

(i) How many different seating arrangements of the 16 prefects are possible? **1**

(ii) If 4 girls and 4 boys were to be chosen at random to fill the back row, how many different groups of 8 can be chosen to fill the back row? **2**

(iii) The middle two seats of the front row were to be occupied by the girl school captain and the boy school captain. If the remaining seats in this row were to be filled by 3 girls and 3 boys chosen at random from the 14 remaining prefects, how many possible arrangements for front row seating are there? **2**

QUESTION 4. *Start a new answer booklet.*

(a) Find the exact value of: $\tan\left[\cos^{-1}\left(\frac{4}{\sqrt{21}}\right)\right]$ **2**

(b) (i) Write down the expansion for: $\tan(\alpha + \beta)$ **1**

(ii) Use the result in (i) above to evaluate, in exact form: **2**

$$\tan^{-1}\left(2\sqrt{2} - 3\right) + \tan^{-1}\left(\sqrt{2}\right)$$

(c) For the function $f(x) = 3\sin^{-1}(3 - 2x)$

(i) Draw a neat sketch of the graph of the function. **3**

(ii) Find the derivative of the function. **2**

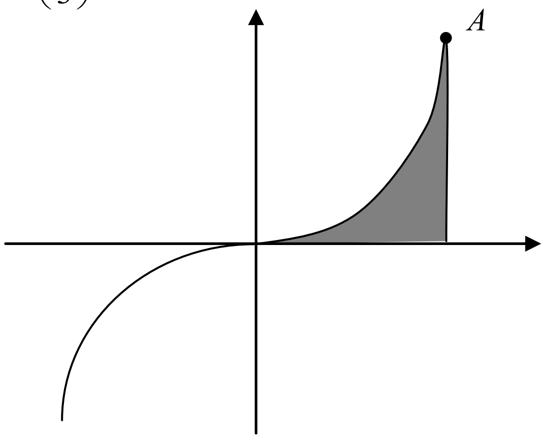
Marks

QUESTION 5. *Start a new answer booklet.*

(a) Find $\int \frac{1}{\sqrt{9-x^2}}\,dx$. **1**

(b) Use the substitution $u = x^2 + 4x - 3$ to evaluate $\int_1^2 \frac{x+2}{\sqrt{x^2+4x-3}}\,dx$ **3**

(c) The graph of $y = 2\sin^{-1}\left(\frac{x}{3}\right)$ is shown below



(i) Write down the coordinates of point *A*. **2**

(ii) Differentiate $y = 2x\sin^{-1}\left(\frac{x}{3}\right) + 2\sqrt{9-x^2}$ **2**

(iii) Hence, or otherwise, find the shaded area. **2**

QUESTION 6. *Start a new answer booklet.*

(a) Solve the inequality $\frac{2}{x} < 1$ **2**

(b) (i) Express $\sqrt{3}\cos x - \sin x$ in the form $r\cos(x+\alpha)$, where $r > 0$ and α is in radians. Justify your answer. **2**

(ii) What is the maximum value of $\sqrt{3}\cos x - \sin x$ and the smallest positive value of x for which it occurs? **2**

(c) Using the substitution $t = \tan\frac{x}{2}$, find the general solution of **4**

$$3\sin x - 2\cos x = 2.$$

Marks

QUESTION 7. *Start a new answer booklet.*

(a) One root of the equation $e^x - x - 2 = 0$ lies between $x = 1$ and $x = 2$. Use one application of Newton's method, with a starting value of $x = 1.5$, to approximate the root to two decimal places. **2**

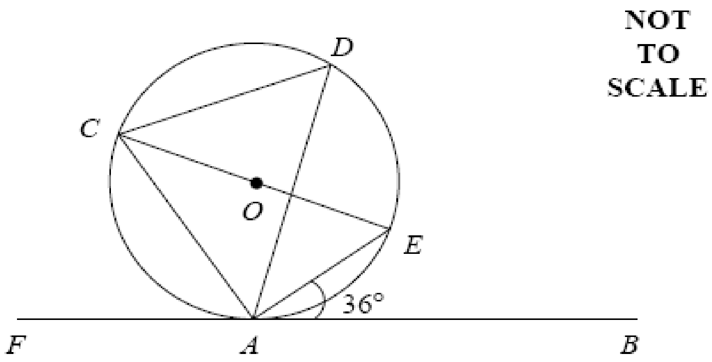
(b) John considered the curve $y = \dfrac{x}{\log_e\left(x^2\right)}$

(i) John was about to change $\log_e\left(x^2\right)$ to $2\log_e x$, but then realised this would actually alter the graph itself. Briefly explain why. **1**

(ii) Accurately describe the domain. **2**

(iii) Find the derivative of the function. **2**

(c)



FB is a tangent meeting a circle at *A*. CE is a diameter, *O* is the centre and *D* lies on the circumference. $\angle BAE = 36°$.
O is the centre and *D* lies on the circumference. $\angle BAE = 36°$.

(i) Find the size of $\angle ACE$, giving reasons. **1**

(ii) Find the size of $\angle ADC$, giving reasons. **2**

Marks

QUESTION 8. *Start a new answer booklet.*

(a) When $x^3 - 3x^2 - ax + 2$ is divided by $x + 3$, the remainder is 4. Find the value of a. **2**

(b) Sketch the curve $y = (3 - x)(x + 1)^2$ **1**
(it is not necessary to find stationary points)

(c) $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are two points on the parabola $x^2 = 4ay$. R is the point of intersection of the tangents to the parabola at P and Q.

(i) Show that the equation of the tangent to the parabola at P is given by: **1**
$px - y - ap^2 = 0$. You may assume that the gradient of the tangent is p.

(ii) Show that the tangents to the parabola at P and Q intersect at the point **2**
$R = (a(p+q), apq)$.

(iii) It is given that the equation of the chord PQ is **1**

$$y = \frac{(p+q)}{2}x - apq. \qquad \text{(DO NOT PROVE THIS)}$$

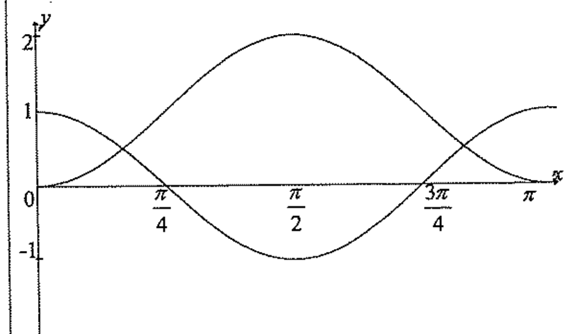
Point T is the intersection of the chord and the axis of the parabola.
Show that T is the point $(0, -apq)$.

(iv) If R is on both the axis of the parabola and the directrix, show that triangle PTR is an isosceles right angled triangle. **3**

Year 12	Mathematics Extension 1	Trial HSC Examination 2010
Question 1	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

PE2 Uses multi-step deductive reasoning in a variety of contexts
PE6 Makes comprehensive use of mathematical language, diagrams and notation for communicating in a wide variety of situations
H5 Applies appropriate techniques from the study of calculus & trigonometry
HE7 Evaluates mathematical solutions to problems and communicates them in an appropriate form

Part	Solutions	Marking Guidelines
PE2	a) (a) Internal point of division, ratio of 3 : 2 Point of division of $(4,-1),(-3,2)$ is $\left(\frac{lx_1+kx_2}{k+l},\frac{ly_1+ky_2}{k+l}\right)=\left(\frac{2\times4+3\times-3}{3+2},\frac{2\times-1+3\times2}{3+2}\right)$ $=\left(\frac{-1}{5},\frac{4}{5}\right)$	2 marks : correct answer 1 mark : significant progress towards answer
HE7	b) (i) $\cos 2x=\cos(x+x)$ $=\cos x\cos x-\sin x\sin x$ $=\cos^2 x-\sin^2 x$ $=1-\sin^2 x-\sin^2 x$ $=1-2\sin^2 x$	1 mark : correct solution
HE7	(ii) Graph of $y=\cos 2x$ has period $\frac{2\pi}{2}=\pi$, $\therefore$ for $0\le x\le\pi$, one wavelength. Amplitude 1. Using $\cos 2x=1-2\sin^2 x$, $2\sin^2 x=1-\cos 2x$ Graph of $y=2\sin^2 x$ is same graph as $y=1-\cos 2x$. i.e. $y=-\cos 2x$, shifted up 1 unit. 	2 marks : both graphs correct 1 mark : one graph correct
HE7	(iii) $y=2\sin^2 x$ and $y=\cos 2x$ meet when $2\sin^2 x=\cos 2x$. i.e. when $2\sin^2 x=1-2\sin^2 x$ Solving $4\sin^2 x=1$ $\sin^2 x=\frac{1}{4}$, $\sin x=\pm\frac{1}{2}$ As solving for $0\le x\le\pi$, x is in quadrants 1, 2 $x=\frac{\pi}{6}$, $\pi-\frac{\pi}{6}$ = $\frac{\pi}{6}$, $\frac{5\pi}{6}$	2 marks : correct answers 1 mark : significant progress towards correct solution
H5, PE2	(iv) For $y=\cos 2x$, $y'=-2\sin 2x$. When $x=\frac{\pi}{6}$, $y'=-2\sin\frac{\pi}{3}=-\sqrt{3}$ For $y=2\sin^2 x$, $y'=4(\sin x)^1\cos x$ $=2\sin 2x$ When $x=\frac{\pi}{6}$, $y'=2\sin\frac{\pi}{3}=\sqrt{3}$. If θ is the angle between the two curves, using $\tan\theta=\left\|\frac{m_1-m_2}{1+m_1m_2}\right\|$, $\tan\theta=\left\|\frac{\sqrt{3}-(-\sqrt{3})}{1+\sqrt{3}\times(-\sqrt{3})}\right\|$ $\therefore\tan\theta=\left\|\frac{2\sqrt{3}}{1-3}\right\|=\left\|-\sqrt{3}\right\|$ $\therefore\tan\theta=\sqrt{3}$ $\theta=\frac{\pi}{3}$.	3 marks : correct solution 2 marks : substantial progress towards correct solution 1 mark : significant progress towards correct solution

Year 12 Question No. 2	Mathematics Extension 1 Solutions and Marking Guidelines	HSC assessment Task 4 2010

Outcomes Addressed in this Question

HE3 uses a variety of strategies to investigate mathematical models of situations involving binomial probability, projectiles, simple harmonic motion, or exponential growth and decay

Outcome	Solutions	Marking Guidelines
	(*i*) $T = P + Ae^{kt}$ $Ae^{kt} = T - P$ $\frac{dT}{dt} = kAe^{kt}$ $\frac{dT}{dt} = k(T-P)$	1 mark correct answer
	(*ii*) $P = 23°, (T = 100, t = 0), (T = 93, t = 2)$ $100 = 23 + Ae^0, A = 77$ $T = 23 + 77e^{kt}$ $93 = 23 + 77e^{2k}$ $k \approx -0 \cdot 0476....$	2 marks correct method leading to correct answer 1 mark substantially correct solution
	(*iii*) $80 = 23 + 77e^{-0.0476t}$ $\frac{80-23}{77} = e^{-0.0476t}$ $t = \ln\left(\frac{80-23}{77}\right) \div (-0 \cdot 0476)$ $t = 6 \cdot 311...\text{min}$ $t \approx 6\text{ min}$	2 marks correct method leading to correct answer 1 mark substantially correct solution
	(b)	
	(*i*) $x = 3\cos\left(2t + \frac{\pi}{3}\right)$ $\dot{x} = \frac{dx}{dt} = -3\sin\left(2t + \frac{\pi}{3}\right) \times 2$ $\dot{x} = \frac{dx}{dt} = -6\sin\left(2t + \frac{\pi}{3}\right)$ $\ddot{x} = \frac{d^2x}{dt^2} = -6\cos\left(2t + \frac{\pi}{3}\right) \times 2$ $\ddot{x} = \frac{d^2x}{dt^2} = -4 \times 3\cos\left(2t + \frac{\pi}{3}\right) = -4x$ Since acceleration obeys the law $\ddot{x} = -n^2x$ the motion is simple harmonic.	3 marks correct method leading to correct answer 2 marks correct differentials 1 mark substantially correct solution
	(ii) $\dot{x} = v = -6\sin\left(2t + \frac{\pi}{3}\right)$ since $-1 \le \sin\left(2t + \frac{\pi}{3}\right) \le 1$ maximum speed $= 6\,cm/s$ occurs when $\left(2t + \frac{\pi}{3}\right) = \frac{\pi}{2}$ $2t = \frac{\pi}{2} - \frac{\pi}{3}$ $2t = \frac{\pi}{6}$ $t = \frac{\pi}{12}$ $x = 3\cos\left(2 \times \frac{\pi}{12} + \frac{\pi}{3}\right)$ $x = 3\cos\left(\frac{\pi}{2}\right), 3\cos\left(2\pi + \frac{\pi}{2}\right), ...$ $x = 0$ in all cases $\therefore$ maximum speed is 6cm/s when $x = 0$.	2 marks total 1 mark For each of distance and velocity correct solution

Year 12 Mathematics Extension 1 Half Yearly Examination 2010

Question No. 3 Solutions and Marking Guidelines

Outcomes Addressed in this Question

HE2 uses inductive reasoning in the construction of proofs

PE3 solves problems involving permutations and combinations.

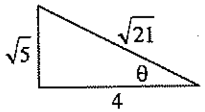
Outcome	Solutions	Marking Guidelines
HE2	**(a)** Since $6^k - 1$ is divisible by 5, Let $6^k - 1 = 5M$ where M is an integer. ie. $6^k = 5M + 1$ Now, $6^{k+1} - 1 = 6 \times 6^k - 1$ $= 6(5M+1) - 1$ $= 30M + 6 - 1$ $= 30M + 5$ $= 5(6M+1)$ which is divisible by 5, as required.	**2 marks** Correct solution **1 mark** Substantial progress towards correct solution.
HE2	**(b)** $\sum_{r=1}^{n} r.2^r = (n-1).2^{n+1} + 2$ 1. Prove true for $n = 1$ $LHS = \sum_{r=1}^{1} r.2^r = 1 \times 2^1 = 2$ $RHS = (1-1) \times 2^{1+1} + 2 = 0 + 2 = 2 = LHS$ $\therefore$ True for n=1 2. Assume true for $n = k$ ie. Assume $\sum_{r=1}^{k} r.2^r = (k-1).2^{k+1} + 2$ Prove true for $n = k+1$ ie. Prove $\sum_{r=1}^{k+1} r.2^r = k.2^{k+2} + 2$ $LHS = \sum_{r=1}^{k+1} r.2^r$ $= \sum_{r=1}^{k} r.2^r + (k+1).2^{k+1}$ $= (k-1)2^{k+1} + 2 + (k+1).2^{k+1}$ $= 2k.2^{k+1} + 2$ $= k.2^1.2^{k+1} + 2$ $= k.2^{k+2} + 2$ $= RHS$ $\therefore$ True for $n = k+1$ 3. If the result is true for $n = k$ it is also true for $n = k+1$ Since the tresult is true for $n = 1$ it is also true for $n = 1 + 1 = 2$, $n = 2 + 1 = 3$, etc. Hence, by the principle of mathematical induction, the result is true for all positive integral values of n.	**3 marks** Correct solution showing clear and logical progression through to conclusion. **2 marks** Substantially correct solution which is not complete through to conclusion or omits steps required to fully justify conclusion. **1 mark** Some progress towards solution, including showing the result true for $n = 1$ as a minimum.
PE3	**(c) (i)** No. of arrangements $= 16!$ $\approx 2.09 \times 10^{13}$	**1 mark** Correct answer.
PE3	**(ii)** No. of groups of 8 $= {}^8C_4 \times {}^8C_4$ $= 70 \times 70$ $= 4900$ [4 girl prefects chosen from 8 × 4 boy prefects chosen from 8]	**2 marks** Correct solution, including evaluation of combinations or any other method. **1 mark** Partially correct solution.
PE3	**(iii)** No. of front row arrangements $= 2 \times {}^7C_3 \times {}^7C_3 \times 6!$ $= 2 \times 35 \times 35 \times 720$ $= 1764000$ [Captains arranged in 2 ways × 3 more girl prefects chosen from the 7 remaining × 3 more boy prefects chosen from the 7 remaining × 6! ways that the prefects (oot captains) can be seated.]	**2 marks** Correct solution, including evaluation of combinations or any other method. **1 mark** Partially correct solution.

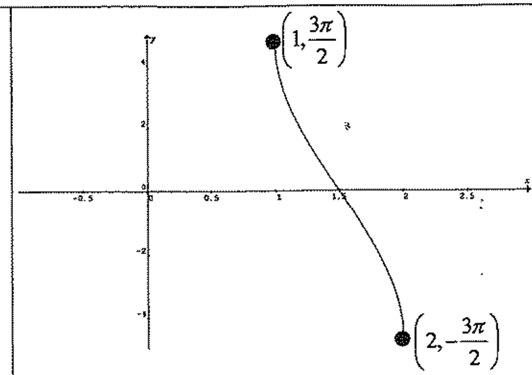
Year 12 Mathematics Extension 1 Half Yearly Examination 2010

Question No. 4 Solutions and Marking Guidelines

Outcomes Addressed in this Question

HE4 uses the relationship between functions, inverse functions and their derivatives

Outcome	Solutions	Marking Guidelines
HE4	(a) Let $\theta = \cos^{-1}\left(\frac{4}{\sqrt{21}}\right)$ $\therefore \cos\theta = \frac{4}{\sqrt{21}}$ On a diagram:  $\tan\left(\cos^{-1}\frac{4}{\sqrt{21}}\right) = \tan\theta$ $= \frac{\sqrt{5}}{4}$	**2 marks** Correct solution **1 mark** Substantially correct solution, demonstrating a knowledge of the meaning of inverse trig. functions.
HE4	(b) (i) $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$	**1 mark** Correct expansion given.
HE4	(ii) Let $\alpha = \tan^{-1}\left(2\sqrt{2}-3\right)$ and $\beta = \tan^{-1}\left(\sqrt{2}\right)$ $\therefore \tan\alpha = 2\sqrt{2}-3$ and $\tan\beta = \sqrt{2}$ Now, $\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$ $= \frac{2\sqrt{2}-3+\sqrt{2}}{1-\left(2\sqrt{2}-3\right)\sqrt{2}}$ $= \frac{3\sqrt{2}-3}{1-4+3\sqrt{2}}$ $= \frac{3\sqrt{2}-3}{3\sqrt{2}-3}$ $= 1$ $\therefore (\alpha+\beta) = \tan^{-1}(1)$ $= \frac{\pi}{4}$ ie. $\tan^{-1}\left(2\sqrt{2}-3c\right) + \tan^{-1}\left(\sqrt{2}\right) = \frac{\pi}{4}$	**2 marks** Correct solution. **1 mark** Substantial progress towards correct solution including demonstratinn of knowledge of the relationship between the trig. function and its inverse.
HE4	(c) $f(x) = 3\sin^{-1}(3-2x)$ Domain: $-1 \le (3-2x) \le 1$ $-4 \le -2x \le -2$ $2 \ge x \ge 1$ Range: $-\frac{\pi}{2} \le \sin^{-1}(3-2x) \le \frac{\pi}{2}$ $-\frac{3\pi}{2} \le 3\sin^{-1}(3-2x) \le \frac{3\pi}{2}$ See graph on following page.	**3 marks** Correctly drawn graph showing correct domain and range and correct orientation. **2 marks** Substantially correct graph with single error in domain, range or orientation. **1 mark** Demonstrates some significant knowledge of how inverse trig. graphs are drawn.

Outcome	Solutions	Marking Guidelines
		
HE4	(ii) $y = 3\sin^{-1}(3-2x)$ Let $u = 3-2x$ Now, $y = 3\sin^{-1}u$ $\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$ $= \frac{3}{\sqrt{1-u^2}}.(-2)$ $= \frac{-6}{\sqrt{1-(3-2x)^2}}$ since $u = 3-2x$ $= \frac{-6}{\sqrt{1-(9-12x+4x^2}}$ $= \frac{-6}{\sqrt{12x-4x^2-8}}$	**2 marks** Correct solntion. **1 mark** Substantial progress towards correct solution, demonstrating some knowledge of differentiating inverse trig. functions or function of a function rule.

Year 12 Question No. 5	Mathematics Extension 1 Solutions and Marking Guidelines	HSC assessment task 4 2010

Outcomes Addressed in this Question

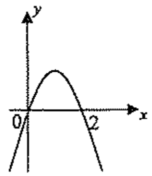
PE5 Determines derivatives which require the application of more than one rule of differentiation
HE6 Determines integrals by reduction to standard form through a given substitution
H8 Uses techniques of integration to calculate areas and volumes
HE4 Uses the relationship between functions, inverse functions and their derivatives.

Part	Solution	Marking Guidelines
a)	$\int \frac{1}{\sqrt{9-x^2}}\,dx = \sin^{-1}\frac{x}{3} + c$	**1 mark** – correct answer
b)	$\int_1^2 \frac{x+2}{\sqrt{x^2+4x-3}}\,dx = \int_2^9 \frac{du}{2\sqrt{u}}$ $= \frac{1}{2}\int_2^9 u^{-\frac{1}{2}}\,du$ $= \frac{1}{2}\left[\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right]_2^9$ $= 3-\sqrt{2}$	**3 marks** – correct solution **2 marks** – substantial progress towards correct solution **1 mark** – some progress towards correct solution
c) i)	$A(3,\pi)$	**2 marks** – correct coordinates **1 mark** – correct value for x or y
b) ii)	$\frac{dy}{dx} = \frac{2x}{3}\frac{3}{\sqrt{9-x^2}} + \sin^{-1}\left(\frac{x}{3}\right).2 + 2.\frac{1}{2}(9-x^2)^{-\frac{1}{2}}.-2x$ $= \frac{2x}{\sqrt{9-x^2}} + 2\sin^{-1}\left(\frac{x}{3}\right) - \frac{2x}{\sqrt{9-x^2}}$ $= 2\sin^{-1}\left(\frac{x}{3}\right)$	**2 marks** – correct solution **1 mark** – substantial progress towards correct solution
c) iii)	$\int_0^3 2\sin^{-1}\left(\frac{x}{3}\right)dx = \left[2x\sin^{-1}\left(\frac{x}{3}\right) + 2\sqrt{9-x^2}\right]_0^3$ $= \left(6\sin^{-1}(1) + 2\sqrt{9-9}\right) - \left(0 + 2\sqrt{9}\right)$ $= (3\pi - 6)$ *square units*	**2 marks** – correct solution **1 mark** – substantial progress towards correct solution

Year 12	Mathematics Extension 1	Trial HSC Examination 2010
Question 6	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

PE3 solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations
HE7 evaluates mathematical solutions to problems and communicates them in an appropriate form

Part	Solutions	Marking Guidelines
PE3	a) $\frac{2}{x} < 1$ $\frac{2}{x} \times x^2 < 1 \times x^2$ (multiplying by positive, $x \neq 0$) $2x - x^2 < 0$ $x(2-x) < 0$ $\therefore x < 0$ and $x > 2$ 	2 marks : correct solution 1 mark : substantial progress towards correct solution
HE7	b) (i) $\sqrt{3}\cos x - \sin x \equiv r\cos(x+\alpha)$ $\equiv r(\cos x\cos\alpha - \sin x\sin\alpha)$ where $r = \sqrt{(\sqrt{3})^2 + (-1)^2} = 2$ $\therefore \sqrt{3}\cos x - \sin x \equiv 2\cos x\cos\alpha - 2\sin x\sin\alpha$ Equating like coefficients, $\sqrt{3} = 2\cos\alpha$, $-1 = -2\sin\alpha$ $\therefore \cos\alpha = \frac{\sqrt{3}}{2}$, $\sin\alpha = \frac{1}{2}$ As sin positive quadrants 1, 2 & cos positive in quadrants 1, 4 α is in quadrant 1. $\therefore \alpha = \frac{\pi}{6}$ $\therefore \sqrt{3}\cos x - \sin x \equiv 2\cos\left(x + \frac{\pi}{6}\right)$	2 marks : correct solution 1 mark : correct value for r or α
HE7	(ii) Maximum value of $\sqrt{3}\cos x - \sin x$ is when $2\cos\left(x+\frac{\pi}{6}\right)$ is a maximum which occurs when $\cos\left(x+\frac{\pi}{6}\right) = 1$. $\therefore$ maximum value is $2 \times 1 = 2$. $\cos\left(x+\frac{\pi}{6}\right) = 1$ when $x + \frac{\pi}{6} = 0, 2\pi, \ldots$ i.e. when $x = \frac{-\pi}{6}, 2\pi - \frac{\pi}{6}, \ldots$ $\therefore$ maximum value is 2, and the smallest positive value of x for which it occurs is $\frac{11\pi}{6}$.	2 marks : correct answers 1 mark : one correct answer or equivalent

HE7	c) If $3\sin x - 2\cos x = 2$ and $t = \tan\frac{x}{2}$, $3 \times \frac{2t}{1+t^2} - 2 \times \frac{1-t^2}{1+t^2} = 2$ $6t - 2(1-t^2) = 2(1+t^2)$ $6t - 2 + 2t^2 = 2 + 2t^2$ $6t = 4$ $t = \frac{2}{3}$ $\therefore \tan\frac{x}{2} = \frac{2}{3}$ $\therefore \frac{x}{2} = n\pi + \tan^{-1}\frac{2}{3}$ Testing $x = \pi$ as a solution to $3\sin x - 2\cos x = 2$: $3\sin\pi - 2\cos\pi = 0 - 2(-1) = 2$ $\therefore$ true for $x = \pi$ $\therefore x = 2n\pi + 2\tan^{-1}\frac{2}{3}$ and $x = (2n+1)\pi$ for any integer n	4 marks : correct solution 3 marks : substantial progress towards correct solution 2 marks : significant progress towards correct solution 1 mark : correct use of t results to solve equation or correct use of general solution to solve equation

Year 12 Question No. 7	Mathematics Extension 1 Solutions and Marking Guidelines	HSC assessment task 4 2010

Outcomes Addressed in this Question

PE3 Solves problems involving circle geometry
PE6 Makes comprehensive use of mathematical language, diagrams and notation for communicating in a wide variety of situations

	a) $f(x)=e^x-x-2$ $f'(x)=e^x-1$ $x_1=1.5-\frac{e^{1.5}-1.5-2}{e^{1.5}-1}$ $=1.5-\frac{0.98168}{3.48168}$ ≈ 1.22	**<u>2 marks</u>** – correct solution **<u>1 mark</u>** – substantial progress towards correct solution
	b) i) The domain of $\log(x^2)$ is all real x, $x\neq 0$. The domain of $2\log x$ is x>0. Changing the function restricts the domain, so half the graph would be lost.	**<u>1 mark</u>** – correct explanation
	b) ii) All real x, $x\neq 0$ and $x\neq \pm 1$	**<u>2 marks</u>** – correct domain **<u>1 mark</u>** – either $x\neq 0$ or $x\neq\pm 1$
	b) iii) $\frac{dy}{dx}=\frac{\log_e(x^2).1-x.\frac{2x}{x^2}}{(\log(x^2))^2}$ $=\frac{\log_e(x^2)-2}{(\log(x^2))^2}$	**<u>2 marks</u>** – correct solution **<u>1 mark</u>** – substantial progress towards correct solution
	c) i) $\angle ACE=36°$ (alternate segment theorem)	**<u>1 mark</u>** – correct answer with correct reason
	c) ii) $\angle CAE=90°$ (angle in a semicircle) $\angle CEA+90°+36°=180°$ (angle sum of ΔACE) $\angle CEA=54°$ $\therefore \angle ADC=54°$ (angles on same arc)	**<u>2 marks</u>** – correct answer with correct reasons **<u>1 mark</u>** – substantial progress towards correct solution

Year 12	Mathematics Extension 1	TRIAL Exam 2010
Question No. 8	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

PE3 solves problems involving polynomials and parametric representations

PE4 uses the parametric representation together with differentiation to identify geometric properties of parabolas

	Solutions	Marking Guidelines

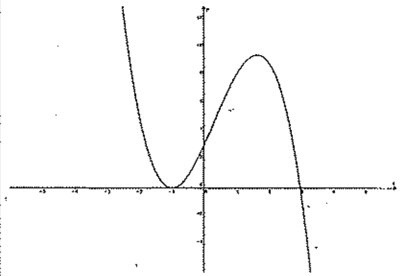
(a) PE3

$$P(x) = x^3 - 3x^2 - ax + 2$$
$$P(-3) = (-3)^3 - 3(-3)^2 - a(-3) + 2 = 4$$
$$4 = -27 - 27 + 3a + 2$$
$$56 = 3a$$
$$a = \frac{56}{3}$$

2 marks – correct solution

1 mark – substantially correct solution

(b)



1 mark – correct solution

(c) (i) PE3

$P(2ap, ap^2)$, $m = p$

Equation of tangent is...

$$y - y_1 = m(x - x_1)$$
$$y - ap^2 = p(x - 2ap)$$
$$y - ap^2 = px - 2ap^2$$
$$px - y - ap^2 = 0$$

1 mark – correct solution

© (ii) PE3

tangent at P is $y = px - ap^2$

tangent at R is $y = qx - aq^2$

solve simult $px - ap^2 = qx - aq^2$

$$px - qx = ap^2 - aq^2$$
$$x(p-q) = a(p-q)(p+q)$$
$$x = a(p+q)$$
$$y = p\left[a(p+q)\right] - ap^2$$
$$= apq$$

so $R = (a(p+q), apq)$

2 marks – correct solution

1 mark – substantially correct solution

(c) (iii) PE3

axis of parabola is $x = 0$ (1)

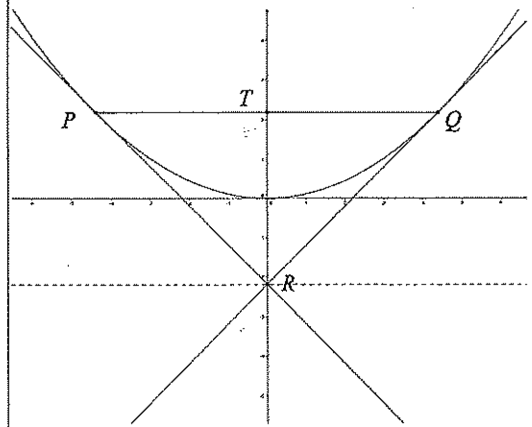
chord PQ is $y = \frac{(p+q)x}{2} - apq$ (2)

sub (1) into (2) $y = 0 - apq$

so $T = (0, -apq)$

1 mark – correct solution

(c) (iv) PE4



$R = (a(p+q), apq)$ is...

on directrix, $y = -a$ and on axis, $x = 0$

ie $apq = -a$ ie $a(p+q) = 0$

$pq = -1$ ...(1) $p + q = 0$, as $x \neq 0$

$p = -q$ ...(2)

$\therefore p, q = \pm 1$

$P(2ap, ap^2) \Rightarrow P(2ap, a)$

$Q(2aq, aq^2) \Rightarrow Q(-2ap, a)$

$R(a(p+q), apq) \Rightarrow R(0, -a)$

$T(0, -apq) \Rightarrow T(0, a)$

$m_{PQ} = \frac{ap^2 - ap^2}{2ap - (-2ap)} = 0$

RT is vertiacal (axis)

$\therefore PQ \perp RT$

ie ΔPTR is right angled

distance $PT = 2a$ $(p = \pm 1)$

distance $TR = 2a$

ie ΔPTR is isosceles

So ΔPTR is an isosceles right angled triangle.

3 marks – correct solution

2 marks – substantially correct solution

1 mark – partial progress *towards* correct solution